

Euclidean Geometry A Guided Inquiry Approach Msri Mathematical Circles Library

This book is a self-contained introduction to the fundamental and interesting topics of geometry. The topics covered include the definitions and properties of 2D and 3D shapes, the basic concepts and formulas of perimeter, area and volume, the Pick's formula, the Cavalieri's principles, the basic geometric transformations, the concepts of tessellations, the Euler's polyhedral formula, and the methods of construction of regular polygons and regular polyhedra. Complemented by diagrams, pictures, examples and exercises, the materials covered are suitable for an introductory course on geometry and will appeal to school students, teachers, junior undergraduate students, mathematics instructors or lecturers alike.

Manifolds, the higher-dimensional analogs of smooth curves and surfaces, are fundamental objects in modern mathematics. Combining aspects of algebra, topology, and analysis, manifolds have also been applied to classical mechanics, general relativity, and quantum field theory. In this streamlined introduction to the subject, the theory of manifolds is presented with the aim of helping the reader achieve a rapid mastery of the essential topics. By the end of the book the reader should be able to compute, at least for simple spaces, one of the most basic topological invariants of a manifold, its de Rham cohomology. Along the way, the reader acquires the knowledge and skills necessary for further study of geometry and topology. The requisite point-set topology is included in an appendix of twenty pages; other appendices review facts from real analysis and linear algebra. Hints and solutions are provided to many of the exercises and problems. This work may be used as the text for a one-semester graduate or advanced undergraduate course, as well as by students engaged in self-study. Requiring only minimal undergraduate prerequisites, 'Introduction to Manifolds' is also an excellent foundation for Springer's GTM 82, 'Differential Forms in Algebraic Topology'.

The aim of this volume is to explain the differences between research-level mathematics and the maths taught at school. Most differences are philosophical and the first few chapters are about general aspects of mathematical thought.

Features the classical themes of geometry with plentiful applications in mathematics, education, engineering, and science Accessible and reader-friendly, Classical Geometry: Euclidean, Transformational, Inversive, and Projective introduces readers to a valuable discipline that is crucial to understanding both spatial relationships and logical reasoning. Focusing on the development of geometric intuition while avoiding the axiomatic method, a problem solving approach is encouraged throughout. The book is strategically divided into three sections: Part One focuses on Euclidean geometry, which provides the foundation for the rest of the material covered throughout; Part Two discusses Euclidean transformations of the plane, as well as groups and their use in studying transformations; and Part Three covers inversive and projective geometry as natural extensions of Euclidean geometry. In addition to featuring real-world applications throughout, Classical Geometry: Euclidean, Transformational, Inversive, and Projective includes: Multiple entertaining and elegant geometry problems at the end of each section for every level of study Fully worked examples with exercises to facilitate comprehension and retention Unique topical coverage, such as the theorems of Ceva and Menelaus and their applications An approach that prepares readers for the art of logical reasoning, modeling, and proofs The book is an excellent textbook for courses in introductory geometry, elementary geometry, modern geometry, and history of mathematics at the undergraduate level for mathematics majors, as well as for engineering and secondary education majors. The book is also ideal for anyone who would like to learn the various applications of elementary geometry.

This is the first major study in any language on J.G. Fichte's philosophy of mathematics and theory of geometry. It investigates both the external formal and internal cognitive parallels between the axioms, intuitions and constructions of geometry and the scientific methodology of the Fichtean system of philosophy. In contrast to "ordinary" Euclidean geometry, in his "Erlanger Logik" of 1805 Fichte posits a model of an "ursprungliche" or original geometry - that is to say, a synthetic and constructivistic conception grounded in ideal archetypal elements that are grasped through geometrical or intelligible intuition. Accordingly, this study classifies Fichte's philosophy of mathematics as a whole as a species of mathematical Platonism or neo-Platonism, and concludes that the "Wissenschaftslehre" itself may be read as an attempt at a new philosophical mathesis, or "mathesis of the mind." "This work testifies to the author's exact and extensive knowledge of the Fichtean texts, as well as of the philosophical, scientific and historical contexts. Wood has opened up completely new paths for Fichte research, and examines with clarity and precision a domain that up to now has hardly been researched." Professor Dr. Marco Ivaldo (University of Naples) "This study, written in a language distinguished by its limpidity and precision, and constantly supported by a close reading of the Fichtean texts and secondary literature, furnishes highly detailed and convincing demonstrations. In directly confronting the difficult historical relationship between the "Wissenschaftslehre" and mathematics, the author has broken new ground that is at once stimulating, decidedly innovative, and elegantly audacious." Professor Dr. Emmanuel Cattin (Universite Blaise-Pascal, Clermont-Ferrand)

The volume begins with an overview of POGIL and a discussion of the science education reform context in which it was developed. Next, cognitive models that serve as the basis for POGIL are presented, including Johnstone's Information Processing Model and a novel extension of it. Adoption, facilitation and implementation of POGIL are addressed next. Faculty who have made the transformation from a traditional approach to a POGIL student-centered approach discuss their motivations and implementation processes. Issues related to implementing POGIL in large classes are discussed and possible solutions are provided. Behaviors of a quality facilitator are presented and steps to create a facilitation

plan are outlined. Succeeding chapters describe how POGIL has been successfully implemented in diverse academic settings, including high school and college classrooms, with both science and non-science majors. The challenges for implementation of POGIL are presented, classroom practice is described, and topic selection is addressed. Successful POGIL instruction can incorporate a variety of instructional techniques. Tablet PC's have been used in a POGIL classroom to allow extensive communication between students and instructor. In a POGIL laboratory section, students work in groups to carry out experiments rather than merely verifying previously taught principles. Instructors need to know if students are benefiting from POGIL practices. In the final chapters, assessment of student performance is discussed. The concept of a feedback loop, which can consist of self-analysis, student and peer assessments, and input from other instructors, and its importance in assessment is detailed. Data is provided on POGIL instruction in organic and general chemistry courses at several institutions. POGIL is shown to reduce attrition, improve student learning, and enhance process skills.

The distinctive approach of Henderson and Taimina's volume stimulates readers to develop a broader, deeper, understanding of mathematics through active experience--including discovery, discussion, writing fundamental ideas and learning about the history of those ideas. A series of interesting, challenging problems encourage readers to gather and discuss their reasonings and understanding. The volume provides an understanding of the possible shapes of the physical universe. The authors provide extensive information on historical strands of geometry, straightness on cylinders and cones and hyperbolic planes, triangles and congruencies, area and holonomy, parallel transport, SSS, ASS, SAA, and AAA, parallel postulates, isometries and patterns, dissection theory, square roots, pythagoras and similar triangles, projections of a sphere onto a plane, inversions in circles, projections (models) of hyperbolic planes, trigonometry and duality, 3-spheres and hyperbolic 3-spaces and polyhedra. For mathematics educators and other who need to understand the meaning of geometry.

In recent years geometry seems to have lost large parts of its former central position in mathematics teaching in most countries. However, new trends have begun to counteract this tendency. There is an increasing awareness that geometry plays a key role in mathematics and learning mathematics. Although geometry has been eclipsed in the mathematics curriculum, research in geometry has blossomed as new ideas have arisen from inside mathematics and other disciplines, including computer science. Due to reassessment of the role of geometry, mathematics educators and mathematicians face new challenges. In the present ICMI study, the whole spectrum of teaching and learning of geometry is analysed. Experts from all over the world took part in this study, which was conducted on the basis of recent international research, case studies, and reports on actual school practice. This book will be of particular interest to mathematics educators and mathematicians who are involved in the teaching of geometry at all educational levels, as well as to researchers in mathematics education.

Introductory Chemistry Modules: A Guided Inquiry Approach, Preliminary Edition, is designed to encourage active and collaborative learning in the classroom using POGIL (Process-Oriented Guided-Inquiry Learning). The philosophy behind the POGIL approach is based on the processes involved in learning, including the need for collaboration when attempting to learn and understand difficult concepts. POGIL is designed to improve student retention rates and to teach students to think analytically and collaboratively in teams, like scientists do, rather than teaching them to memorize important conclusions arrived at by great scientists of the past. The 52 activities in this preliminary edition workbook, a sampling of those that will appear in the complete First Edition for Introduction to Chemistry courses, guide students through self-directed explorations designed to facilitate both content mastery and learning skills. By looking carefully at new problems, constructing logical conclusions based on observations, and discussing the merits of their conclusions with peers, students develop a stronger conceptual understanding of and appreciation for the material. The activities include some of the following sections.

This textbook provides a unified and concise exploration of undergraduate mathematics by approaching the subject through its history. Readers will discover the rich tapestry of ideas behind familiar topics from the undergraduate curriculum, such as calculus, algebra, topology, and more. Featuring historical episodes ranging from the Ancient Greeks to Fermat and Descartes, this volume offers a glimpse into the broader context in which these ideas developed, revealing unexpected connections that make this ideal for a senior capstone course. The presentation of previous versions has been refined by omitting the less mainstream topics and inserting new connecting material, allowing instructors to cover the book in a one-semester course. This condensed edition prioritizes succinctness and cohesiveness, and there is a greater emphasis on visual clarity, featuring full color images and high quality 3D models. As in previous editions, a wide array of mathematical topics are covered, from geometry to computation; however, biographical sketches have been omitted. Mathematics and Its History: A Concise Edition is an essential resource for courses or reading programs on the history of mathematics. Knowledge of basic calculus, algebra, geometry, topology, and set theory is assumed. From reviews of previous editions: "Mathematics and Its History is a joy to read. The writing is clear, concise and inviting. The style is very different from a traditional text. I found myself picking it up to read at the expense of my usual late evening thriller or detective novel.... The author has done a wonderful job of tying together the dominant themes of undergraduate mathematics." Richard J. Wilders, MAA, on the Third Edition "The book...is presented in a lively style without unnecessary detail. It is very stimulating and will be appreciated not only by students. Much attention is paid to problems and to the development of mathematics before the end of the nineteenth century.... This book brings to the non-specialist interested in mathematics many interesting results. It can be recommended for seminars and will be enjoyed by the broad mathematical community." European Mathematical Society, on the Second Edition

Teaching Mathematics in Grades 6 - 12 by Randall E. Groth explores how research in mathematics education can inform teaching practice in grades 6-12. The author shows preservice mathematics teachers the value of being a "researcher—constantly experimenting with methods for developing students' mathematical thinking—and connecting this research to practices that enhance students' understanding of the material. Ultimately, preservice teachers will gain a deeper understanding of the types of mathematical knowledge students bring to school, and how students' thinking may develop in response to different teaching strategies.

This book provides an inquiry-based introduction to advanced Euclidean geometry. It utilizes dynamic geometry software, specifically GeoGebra, to explore the statements and proofs of many of the most interesting theorems in the subject. Topics covered include triangle centers, inscribed, circumscribed, and escribed circles, medial and orthic triangles, the nine-point circle, duality,

and the theorems of Ceva and Menelaus, as well as numerous applications of those theorems. The final chapter explores constructions in the Poincaré disk model for hyperbolic geometry. The book can be used either as a computer laboratory manual to supplement an undergraduate course in geometry or as a stand-alone introduction to advanced topics in Euclidean geometry. The text consists almost entirely of exercises (with hints) that guide students as they discover the geometric relationships for themselves. First the ideas are explored at the computer and then those ideas are assembled into a proof of the result under investigation. The goals are for the reader to experience the joy of discovering geometric relationships, to develop a deeper understanding of geometry, and to encourage an appreciation for the beauty of Euclidean geometry.

The first book to present a readable explanation of Gödel's theorem to both scholars and non-specialists, this is a gripping combination of science and accessibility, offering those with a taste for logic and philosophy the chance to satisfy their intellectual curiosity.

Geometry Designed for Understanding Jacobs' Geometry utilizes a clear, conversational, engaging approach to teach your student the concepts, principles, and application of Geometry through practical, real-life application! Harold Jacobs guides your student through Geometry, enabling them to discover the concepts & their applications for themselves in order to develop an understanding of the principles that goes beyond simple memorization to pass a test. Jacobs' unique instructional approach to math means your student: Develops a true understanding of geometric principles Interacts with concepts using real-world examples, ensuring they'll know exactly how to apply the material they are learning to real-life and other academic subjects Is prepared to take their understanding of Geometry concepts outside the math textbook and successfully apply them to higher math courses, sciences, & everyday life Is equipped with an understanding of the foundational mathematical concepts of Geometry—and once a student truly understands the concepts in Geometry, they are equipped & prepared for all higher math & sciences! Engaging, Real-World Instruction Understanding both the why and how of Geometry is foundational to your student's success in high school and college. Jacobs' Geometry provides students with a clear and thorough understanding of why concepts work, as well as how they are applied to solve real-world problems. A Top Choice for High School Success & College Prep Jacobs' Geometry has proven its ability to guide students towards success and is still the choice of top teachers and schools. The unique instructional method within Jacobs' Geometry ensures your student understands both the why and how of Geometry and establishes a strong foundation for higher math & science courses. If your student is planning for college or a STEM career, Jacobs' Geometry ensures they are equipped with the tools they need to succeed! Geometry Student Text Includes: Full Color Illustrations 16 sections, covering deductive reasoning, lines & angles, congruence, inequalities, quadrilaterals, area, triangles, circles, theorems, polygons, geometric solids, and more! Answers to select exercises in the back of the text Flexible based on focus & intensity of course Set I exercises review ideas & concepts from previous lessons to provide ongoing application of material. Set II exercises allow student to apply material from the new lesson Set III exercises provided additional, more challenging problems

This volume is a case study of education reform and innovation using technology that examines the issue from a wide variety of perspectives. It brings together the views and experiences of software designers, curriculum writers, teachers and students, researchers and administrators. Thus, it stands in contrast to other analyses of innovation that tend to look through the particular prisms of research, classroom practice, or software design. The Geometric Supposer encourages a belief in a better tomorrow for schools. On its surface, the Geometric Supposer provides the means for radically altering the way in which geometry is taught and the quality of learning that can be achieved. At a deeper level, however, it suggests a powerful metaphor for improving education that can be played out in many different instructional contexts.

This Festschrift contains numerous colorful and eclectic essays from well-known mathematicians, philosophers, logicians, and linguists celebrating the 90th birthday of Reuben Hersh. The essays offer, in part, attempts to answer the following questions set forth by Reuben himself as a focus for this volume: Can practicing mathematicians, as such, contribute anything to the philosophy of math? Can or should philosophers of math, as such, say anything to practicing mathematicians? Twenty or fifty years from now, what will be similar, and what will, or could, or should be altogether different: About the philosophy of math? About math education? About math research institutions? About data processing and scientific computing? The essays also offer glimpses into Reuben's fertile mind and his lasting influence on the mathematical community, as well as revealing the diverse roots, obstacles and philosophical dispositions that characterize the working lives of mathematicians. With contributions from a veritable "who's who" list of 20th century luminaries from mathematics and philosophy, as well as from Reuben himself, this volume will appeal to a wide variety of readers from curious undergraduates to prominent mathematicians.

The main part of this book describes the first semester of the existence of a successful and now highly popular program for elementary school students at the Berkeley Math Circle. The topics discussed in the book introduce the participants to the basics of many important areas of modern mathematics, including logic, symmetry, probability theory, knot theory, cryptography, fractals, and number theory. Each chapter in the first part of this book consists of two parts. It starts with generously illustrated sets of problems and hands-on activities. This part is addressed to young readers who can try to solve problems on their own or to discuss them with adults. The second part of each chapter is addressed to teachers and parents. It includes comments on the topics of the lesson, relates those topics to discussions in other chapters, and describes the actual reaction of math circle participants to the proposed activities. The supplementary problems that were discussed at workshops of Math Circle at Kansas State University are given in the second part of the book. The book is richly illustrated, which makes it attractive to its young audience. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession. Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

In literary studies today, debates about the purpose of literary criticism and about the place of formalism within it continue to simmer across periods and approaches. Anna Kornbluh contributes to—and substantially shifts—that conversation in *The Order of Forms* by offering an exciting new category, political formalism, which she articulates through the co-emergence of aesthetic and mathematical formalisms in the nineteenth century. Within this framework, criticism can be understood as more affirmative and constructive, articulating commitments to aesthetic expression and social collectivity. Kornbluh offers a powerful argument that political formalism, by valuing forms of sociability like the city and the state in and of themselves, provides a better understanding of literary form and its political possibilities than approaches that view form as a constraint. To make this argument, she takes up the case of literary realism, showing how novels by Dickens, Brontë, Hardy, and Carroll engage mathematical formalism as part of their political imagining. Realism, she shows, is best understood as an exercise in social modeling—more like formalist mathematics than social documentation. By modeling society, the realist novel focuses on what it considers the most elementary features of social relations and generates unique political insights. Proposing both this new theory of realism and the idea of political formalism, this inspired,

eye-opening book will have far-reaching implications in literary studies.

One of the most significant tasks facing mathematics educators is to understand the role of mathematical reasoning and proving in mathematics teaching, so that its presence in instruction can be enhanced. This challenge has been given even greater importance by the assignment to proof of a more prominent place in the mathematics curriculum at all levels. Along with this renewed emphasis, there has been an upsurge in research on the teaching and learning of proof at all grade levels, leading to a re-examination of the role of proof in the curriculum and of its relation to other forms of explanation, illustration and justification. This book, resulting from the 19th ICMI Study, brings together a variety of viewpoints on issues such as: The potential role of reasoning and proof in deepening mathematical understanding in the classroom as it does in mathematical practice. The developmental nature of mathematical reasoning and proof in teaching and learning from the earliest grades. The development of suitable curriculum materials and teacher education programs to support the teaching of proof and proving. The book considers proof and proving as complex but foundational in mathematics. Through the systematic examination of recent research this volume offers new ideas aimed at enhancing the place of proof and proving in our classrooms.

An introduction to computational complexity theory, its connections and interactions with mathematics, and its central role in the natural and social sciences, technology, and philosophy Mathematics and Computation provides a broad, conceptual overview of computational complexity theory—the mathematical study of efficient computation. With important practical applications to computer science and industry, computational complexity theory has evolved into a highly interdisciplinary field, with strong links to most mathematical areas and to a growing number of scientific endeavors. Avi Wigderson takes a sweeping survey of complexity theory, emphasizing the field's insights and challenges. He explains the ideas and motivations leading to key models, notions, and results. In particular, he looks at algorithms and complexity, computations and proofs, randomness and interaction, quantum and arithmetic computation, and cryptography and learning, all as parts of a cohesive whole with numerous cross-influences. Wigderson illustrates the immense breadth of the field, its beauty and richness, and its diverse and growing interactions with other areas of mathematics. He ends with a comprehensive look at the theory of computation, its methodology and aspirations, and the unique and fundamental ways in which it has shaped and will further shape science, technology, and society. For further reading, an extensive bibliography is provided for all topics covered. Mathematics and Computation is useful for undergraduate and graduate students in mathematics, computer science, and related fields, as well as researchers and teachers in these fields. Many parts require little background, and serve as an invitation to newcomers seeking an introduction to the theory of computation. Comprehensive coverage of computational complexity theory, and beyond High-level, intuitive exposition, which brings conceptual clarity to this central and dynamic scientific discipline Historical accounts of the evolution and motivations of central concepts and models A broad view of the theory of computation's influence on science, technology, and society Extensive bibliography

This is a light-hearted introduction to game theory suitable for advanced undergraduate students or beginning graduate students. It answers three questions. What is game theory? How is game theory applied? Why is game theory right?

This text provides a historical perspective on plane geometry and covers non-neutral Euclidean geometry, circles and regular polygons, projective geometry, symmetries, inversions, informal topology, and more. Includes 1,000 practice problems. Solutions available. 2003 edition.

From preeminent math personality and author of *The Joy of x*, a brilliant and endlessly appealing explanation of calculus - how it works and why it makes our lives immeasurably better. Without calculus, we wouldn't have cell phones, TV, GPS, or ultrasound. We wouldn't have unraveled DNA or discovered Neptune or figured out how to put 5,000 songs in your pocket.

Though many of us were scared away from this essential, engrossing subject in high school and college, Steven Strogatz's brilliantly creative, down-to-earth history shows that calculus is not about complexity; it's about simplicity. It harnesses an unreal number--infinity--to tackle real-world problems, breaking them down into easier ones and then reassembling the answers into solutions that feel miraculous. *Infinite Powers* recounts how calculus tantalized and thrilled its inventors, starting with its first glimmers in ancient Greece and bringing us right up to the discovery of gravitational waves (a phenomenon predicted by calculus). Strogatz reveals how this form of math rose to the challenges of each age: how to determine the area of a circle with only sand and a stick; how to explain why Mars goes "backwards" sometimes; how to make electricity with magnets; how to ensure your rocket doesn't miss the moon; how to turn the tide in the fight against AIDS. As Strogatz proves, calculus is truly the language of the universe. By unveiling the principles of that language, *Infinite Powers* makes us marvel at the world anew.

Geometry has been an essential element in the study of mathematics since antiquity. Traditionally, we have also learned formal reasoning by studying Euclidean geometry. In this book, David Clark develops a modern axiomatic approach to this ancient subject, both in content and presentation. Mathematically, Clark has chosen a new set of axioms that draw on a modern understanding of set theory and logic, the real number continuum and measure theory, none of which were available in Euclid's time. The result is a development of the standard content of Euclidean geometry with the mathematical precision of Hilbert's foundations of geometry. In particular, the book covers all the topics listed in the Common Core State Standards for high school synthetic geometry. The presentation uses a guided inquiry, active learning pedagogy. Students benefit from the axiomatic development because they themselves solve the problems and prove the theorems with the instructor serving as a guide and mentor. Students are thereby empowered with the knowledge that they can solve problems on their own without reference to authority. This book, written for an undergraduate axiomatic geometry course, is particularly well suited for future secondary school teachers. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

This book on two-dimensional geometry uses a problem-solving approach to actively engage students in the learning process. The aim is to guide readers through the story of the subject, while giving them room to discover and partially construct the story themselves. The book bridges the study of plane geometry and the study of curves and surfaces of non-constant curvature in three-dimensional Euclidean space. One useful feature is that the book can be adapted to suit different audiences. The first half of the text covers plane geometry without and with Euclid's Fifth Postulate, followed by a brief synthetic treatment of spherical geometry through the excess angle formula. This part only requires a background in high school geometry and basic trigonometry and is suitable for a quarter course for future high school geometry teachers. A brief foray into the second half could complete a semester course. The second half of the text gives a uniform treatment of all the complete, simply connected, two-dimensional geometries of constant curvature, one geometry for each real number (its curvature), including their groups of isometries, geodesics, measures of lengths and areas, as well as formulas for areas of regions bounded by polygons in terms of the curvature of the geometry and the sum of the interior angles of the polygon. A basic knowledge of real linear algebra and calculus of several (real) variables is useful background for this portion of the text.

In this revolutionary book, a renowned computer scientist explains the importance of teaching children the basics of computing and how it can prepare them to succeed in the ever-evolving tech world. Computers have completely changed the way we teach children. We have Mindstorms to thank for that. In this book, pioneering computer scientist Seymour Papert uses the invention of LOGO, the first child-friendly programming language, to make the case for the value of teaching children with computers. Papert argues that children are more than capable of mastering computers, and that teaching computational processes like de-bugging in the classroom can change the way we learn everything else. He also shows that schools saturated with technology can actually improve socialization and interaction among students and between students and teachers. Technology changes every day, but the basic ways that computers can help us learn remain. For thousands of teachers and parents who have sought creative ways to help children learn with computers, Mindstorms is their bible.

This spectacularly clear introduction to abstract algebra is designed to make the study of all required topics and the reading and writing of proofs both accessible and enjoyable for readers encountering the subject for the first time. Number Theory. Groups. Commutative Rings. Modules. Algebras. Principal Idea Domains. Group Theory II. Polynomials In Several Variables. For anyone interested in learning abstract algebra.

How sharks have been depicted over centuries and across cultures—and what sharks see when they look back. We encounter the world through surfaces: the screen, the page, our skin, the ocean's swell. Here on the sea is the surfer, positioned at the edge of the collapsing wave. And lurking underneath in a monstrous mirroring is the shark. When the two meet, carving along the surface, breaking through the boundary, death appears. Steering her analysis from the newspaper obituary through literature and past cinema, Melissa McCarthy investigates a fundamental aspect of the human condition: our state of being between life and death, always in precarious and watery balance. *Sharks, Death, Surfers* observes how sharks have been depicted over centuries and across cultures, then flips the lens (and dissects the cornea) to consider what sharks see when they look back. These refracted lines of inquiry—optical, philosophical, historical—converge at the focal point where we can fix the image of the surfer and the shark. This is the picture McCarthy frames: the cartilaginous companions gliding together in a perfect model of how to read, navigate, and exist.

Many mathematicians have been drawn to mathematics through their experience with math circles: extracurricular programs exposing teenage students to advanced mathematical topics and a myriad of problem solving techniques and inspiring in them a lifelong love for mathematics. Founded in 1998, the Berkeley Math Circle (BMC) is a pioneering model of a U.S. math circle, aspiring to prepare our best young minds for their future roles as mathematics leaders. Over the last decade, 50 instructors—from university professors to high school teachers to business tycoons—have shared their passion for mathematics by delivering more than 320 BMC sessions full of mathematical challenges and wonders. Based on a dozen of these sessions, this book encompasses a wide variety of enticing mathematical topics: from inversion in the plane to circle geometry; from combinatorics to Rubik's cube and abstract algebra; from number theory to mass point theory; from complex numbers to game theory via invariants and monovariants. The treatments of these subjects encompass every significant method of proof and emphasize ways of thinking and reasoning via 100 problem solving techniques. Also featured are 300 problems, ranging from beginner to intermediate level, with occasional peaks of advanced problems and even some open questions. The book presents possible paths to studying mathematics and inevitably falling in love with it, via teaching two important skills: thinking creatively while still "obeying the rules," and making connections between problems, ideas, and theories. The book encourages you to apply the newly acquired knowledge to problems and guides you along the way, but rarely gives you ready answers. "Learning from our own mistakes" often occurs through discussions of non-proofs and common problem solving pitfalls. The reader has to commit to mastering the new theories and techniques by "getting your hands dirty" with the problems, going back and reviewing necessary problem solving techniques and theory, and persistently moving forward in the book. The mathematical world is huge: you'll never know everything, but you'll learn where to find things, how to connect and use them. The rewards will be substantial. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

The story of geometry is the story of mathematics itself: Euclidean geometry was the first branch of mathematics to be systematically studied and placed on a firm logical foundation, and it is the prototype for the axiomatic method that lies at the foundation of modern mathematics. It has been taught to students for more than two millennia as a mode of logical thought. This book tells the story of how the axiomatic method has progressed from Euclid's time to ours, as a way of understanding what mathematics is, how we read and evaluate mathematical arguments, and why mathematics has achieved the level of certainty it has. It is designed primarily for advanced undergraduates who plan to teach secondary school geometry, but it should also provide something of interest to anyone who wishes to understand geometry and the axiomatic method better. It introduces a modern, rigorous, axiomatic treatment of Euclidean and (to a lesser extent) non-Euclidean geometries, offering students ample opportunities to practice reading and writing proofs while at the same time developing most of the concrete geometric relationships that secondary teachers will need to know in the classroom. -- P. [4] of cover.

What are "essential questions," and how do they differ from other kinds of questions? What's so great about them? Why should you design and use essential questions in your classroom? Essential questions (EQs) help target standards as you organize curriculum content into coherent units that yield focused and thoughtful learning. In the classroom, EQs are used to stimulate students' discussions and promote a deeper understanding of the content. Whether you are an Understanding by Design (UbD) devotee or are searching for ways to address standards—local or Common Core State Standards—in an engaging way, Jay McTighe and Grant Wiggins provide practical guidance on how to design, initiate, and embed inquiry-based teaching and learning in your classroom. Offering dozens of examples, the authors explore the usefulness of EQs in all K-12 content areas, including skill-based areas such as math, PE, language instruction, and arts education. As an important element of their backward design approach to designing curriculum, instruction, and assessment, the authors *Give a comprehensive explanation of why EQs are so important; *Explore seven defining characteristics of EQs; *Distinguish between topical and overarching questions and their uses; *Outline the rationale for using EQs as the focal point in creating units of study; and *Show how to create effective EQs, working from sources including standards, desired understandings, and student misconceptions. Using essential questions can be challenging—for both teachers and students—and this book provides guidance through practical and proven processes, as well as suggested "response strategies" to encourage student engagement. Finally, you will learn how to create a culture of inquiry so that all members of the educational community—students, teachers, and administrators—benefit from the increased rigor and deepened understanding that emerge when essential questions become a guiding force for learners of all ages.

A TeXas Style Introduction to Proof is an IBL textbook designed for a one-semester course on proofs (the "bridge course") that also introduces TeX as a tool students can use to communicate their work. As befitting "textless" text, the book is, as one reviewer characterized it, "minimal." Written in an easy-going style, the exposition is just enough to support the activities, and it is clear, concise, and effective. The book is well organized and contains ample carefully selected exercises that are varied, interesting, and probing, without being discouragingly difficult.

A historical introduction to non-Euclidean geometry.

String theory says we live in a ten-dimensional universe, but that only four are accessible to our everyday senses. According to theorists, the missing six are curled up in bizarre structures known as Calabi-Yau manifolds. In *The Shape of Inner Space*, Shing-Tung Yau, the man who mathematically proved that these manifolds exist, argues that not only is geometry fundamental to string theory, it is also fundamental to the very nature of our universe. Time and again, where Yau has gone, physics has followed. Now for the first time, readers will follow Yau's penetrating thinking on where we've been, and where mathematics will take us next. A fascinating exploration of a world we are only just beginning to grasp, *The Shape of Inner Space* will change the way we consider the universe on both its grandest and smallest scales.

This is the second edition of the text *Elementary Real Analysis* originally published by Prentice Hall (Pearson) in 2001. Chapter 1. Real Numbers Chapter 2. Sequences Chapter 3. Infinite sums Chapter 4. Sets of real numbers Chapter 5. Continuous functions Chapter 6. More on continuous functions and sets Chapter 7. Differentiation Chapter 8. The Integral Chapter 9. Sequences and series of functions Chapter 10. Power series Chapter 11. Euclidean Space \mathbb{R}^n Chapter 12. Differentiation on \mathbb{R}^n Chapter 13. Metric Spaces

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