An Introduction To The Fractional Calculus And Fractional Differential Equations

This multi-volume handbook is the most up-to-date and comprehensive reference work in the field of fractional calculus and its numerous applications. This second volume collects authoritative chapters covering the mathematical theory of fractional calculus, including ordinary and partial differential equations of fractional order, inverse problems, and evolution equations.

Local Fractional Integral Transforms and Their Applications provides information on how local fractional calculus has been successfully applied to describe the numerous widespread real-world phenomena in the fields of physical sciences and engineering sciences that involve non-differentiable behaviors. The methods of integral transforms via local fractional calculus have been used to solve various local fractional ordinary and local fractional partial differential equations and also to figure out the presence of the fractal phenomenon. The book presents the basics of the local fractional derivative operators and investigates some new results in the area of local integral transforms. Provides applications of local fractional Fourier Series Discusses definitions for local fractional Laplace transforms Explains local fractional Laplace transforms coupled with analytical methods

An Introduction to Fractional Control outlines the theory, techniques and applications of fractional control.

The fractional Laplacian, also called the Riesz fractional derivative, describes an unusual diffusion process associated with random excursions. The Fractional Laplacian explores applications of the fractional Laplacian in science, engineering, and other areas where long-range interactions and conceptual or physical particle jumps resulting in an irregular diffusive or conductive flux are encountered. Presents the material at a level suitable for a broad audience of scientists and engineers with rudimentary background in ordinary differential equations and integral calculus Clarifies the concept of the fractional Laplacian for functions in one, two, three, or an arbitrary number of dimensions defined over the entire space, satisfying periodicity conditions, or restricted to a finite domain Covers physical and mathematical concepts as well as detailed mathematical derivations Develops a numerical framework for solving differential equations involving the fractional Laplacian and presents specific algorithms accompanied by numerical results in one, two, and three dimensions Discusses viscous flow and physical examples from scientific and engineering disciplines Written by a prolific author well known for his contributions in fluid mechanics, biomechanics, applied mathematics, scientific computing, and computer science, the book emphasizes fundamental ideas and practical numerical computation. It includes original material and novel numerical methods.

Fractional calculus is a collection of relatively little-known mathematical results concerning generalizations of differentiation and integration to noninteger orders. While these results have been accumulated over centuries in various branches of mathematics, they have until recently found little appreciation or application in physics and other mathematically oriented sciences. This situation is beginning to change, and there are now a growing number of research areas in physics which employ fractional calculus. This volume provides an introduction to fractional calculus for physicists, and collects easily accessible review articles surveying those areas of physics in which applications of fractional calculus have recently become prominent.

Contents:An Introduction to Fractional Calculus (P L Butzer & U Westphal)Fractional Time Evolution (R Hilfer)Fractional Powers of Infinitesimal Generators of Semigroups (U Westphal)Fractional Differences, Derivatives and Fractal Time Series (B J West & P Grigolini)Fractional Kinetics of Hamiltonian Chaotic Systems (G M Zaslavsky)Polymer Science Applications of Path-Integration, Integral Equations, and Fractional Calculus (J F

Douglas)Applications to Problems in Polymer Physics and Rheology (H Schiessel et al.)Applications of Fractional Calculus Techniques to Problems in Biophysics (T F Nonnenmacher & R Metzler)Fractional Calculus and Regular Variation in Thermodynamics (R Hilfer) Readership: Statistical, theoretical and mathematical physicists. Keywords:Fractional Calculus in PhysicsReviews: “This monograph provides a systematic treatment of the theory and applications of fractional calculus for physicists. It contains nine review articles surveying those areas in which fractional calculus has become important. All the chapters are self-contained.” Mathematics Abstracts

In the last two decades, fractional (or non integer) differentiation has played a very important role in various fields such as mechanics, electricity, chemistry, biology, economics, control theory and signal and image processing. For example, in the last three fields, some important considerations such as modelling, curve fitting, filtering, pattern recognition, edge detection, identification, stability, controllability, observability and robustness are now linked to long-range dependence phenomena. Similar progress has been made in other fields listed here. The scope of the book is thus to present the state of the art in the study of fractional systems and the application of fractional differentiation. As this volume covers recent applications of fractional calculus, it will be of interest to engineers, scientists, and applied mathematicians.

This book introduces a series of problems and methods insufficiently discussed in the field of Fractional Calculus – a major, emerging tool relevant to all areas of scientific inquiry. The authors present examples based on symbolic computation, written in Maple and Mathematica, and address both mathematical and computational areas in the context of mathematical modeling and the generalization of classical integer-order methods. Distinct from most books, the present volume fills the gap between mathematics and computer fields, and the transition from integer- to fractional-order methods.

The book presents a concise introduction to the basic methods and strategies in fractional calculus which enables the reader to catch up with the state-of-the-art in this field and to participate and contribute in the development of this exciting research area. This book is devoted to the application of fractional calculus on physical problems. The fractional concept is applied to subjects in classical mechanics, image processing, folded potentials in cluster physics, infrared spectroscopy, group theory, quantum mechanics, nuclear physics, hadron spectroscopy up to quantum field theory and will surprise the reader with new intriguing insights. This new, extended edition includes additional chapters about numerical solution of the fractional Schrödinger equation, self-similarity and the geometric interpretation of non-isotropic fractional differential operators. Motivated by the positive response, new exercises with elaborated solutions are added, which significantly support a deeper understanding of the general aspects of the theory. Besides students as well as researchers in this field, this book will also be useful as a supporting medium for teachers teaching courses devoted to this subject.

??? Topics in Fractional Differential Equations is devoted to the existence and uniqueness of solutions for various classes of Darboux problems for hyperbolic differential equations or inclusions involving the Caputo fractional derivative.

??Fractional calculus generalizes the integrals and derivatives to non-integer orders. During the last decade, fractional calculus was found to play a fundamental role in the modeling of a considerable number of phenomena; in particular the modeling of memory-dependent and complex media such as porous media. It has emerged as an important tool for the study of dynamical systems where classical methods reveal strong limitations. Some equations present delays which may be finite, infinite, or state-dependent. Others are subject to an impulsive effect. The above problems are studied using the fixed point approach, the method of upper and lower solution, and the

Kuratowski measure of noncompactness. This book is addressed to a wide audience of specialists such as mathematicians, engineers, biologists, and physicists. ?

Presents a unified treatment of anomalous diffusion problems using fractional calculus in a wide range of applications across scientific and technological disciplines.

In this book, we study theoretical and practical aspects of computing methods for mathematical modelling of nonlinear systems. A number of computing techniques are considered, such as methods of operator approximation with any given accuracy; operator interpolation techniques including a non-Lagrange interpolation; methods of system representation subject to constraints associated with concepts of causality, memory and stationarity; methods of system representation with an accuracy that is the best within a given class of models; methods of covariance matrix estimation; methods for low-rank matrix approximations; hybrid methods based on a combination of iterative procedures and best operator approximation; and methods for information compression and filtering under condition that a filter model should satisfy restrictions associated with causality and different types of memory. As a result, the book represents a blend of new methods in general computational analysis, and specific, but also generic, techniques for study of systems theory ant its particular branches, such as optimal filtering and information compression. - Best operator approximation, - Non-Lagrange interpolation, - Generic Karhunen-Loeve transform - Generalised low-rank matrix approximation - Optimal data compression - Optimal nonlinear filtering

This book focuses on solving practical problems in calculus with MATLAB. Descriptions and sketching of functions and sequences are introduced first, followed by the analytical solutions of limit, differentiation, integral and function approximation problems of univariate and multivariate functions. Advanced topics such as numerical differentiations and integrals, integral transforms as well as fractional calculus are also covered in the book.

Fractional calculus is undergoing rapidly and ongoing development. We can already recognize, that within its framework new concepts and strategies emerge, which lead to new challenging insights and surprising correlations between different branches of physics. This book is an invitation both to the interested student and the professional researcher. It presents a thorough introduction to the basics of fractional calculus and guides the reader directly to the current state-of-the-art physical interpretation. It is also devoted to the application of fractional calculus on physical problems, in the subjects of classical mechanics, friction, damping, oscillations, group theory, quantum mechanics, nuclear physics, and hadron spectroscopy up to quantum field theory.

Fractional order calculus is finding increasing interest in the control system community. Hardware realizations of fractional order controllers have sparked off a renewed zeal into the investigations of control system design in the light of fractional calculus. As such many notions of integer order LTI systems are being modified and extended to incorporate these new concepts. Computational Intelligence (CI) techniques have been applied to engineering problems to find solutions to many hitherto intractable conundrums and is a useful tool for dealing with problems of higher computational complexity. This book borders on the interface between CI techniques and fractional calculus, and looks at ways in which fractional order control systems may be designed or enhanced using CI based paradigms. To the best of the author's knowledge this is the first book of its kind exclusively dedicated to the application of computational

intelligence techniques in fractional order systems and control. The book tries to assimilate various existing concepts in this nascent field of fractional order intelligent control and is aimed at researchers and post graduate students working in this field.
The first derivative of a particle coordinate means its velocity, the second means its acceleration, but what does a fractional order derivative mean? Where does it come from, how does it work, where does it lead to? The two-volume book written on high didactic level answers these questions. Fractional Derivatives for Physicists and Engineers— The first volume contains a clear introduction into such a modern branch of analysis as the fractional calculus. The second develops a wide panorama of applications of the fractional calculus to various physical problems. This book recovers new perspectives in front of the reader dealing with turbulence and semiconductors, plasma and thermodynamics, mechanics and quantum optics, nanophysics and astrophysics. The book is addressed to students, engineers and physicists, specialists in theory of probability and statistics, in mathematical modeling and numerical simulations, to everybody who doesn't wish to stay apart from the new mathematical methods becoming more and more popular. Prof. Vladimir V. UCHAIKIN is a known Russian scientist and pedagogue, a Honored Worker of Russian High School, a member of the Russian Academy of Natural Sciences. He is the author of about three hundreds articles and more than a dozen books (mostly in Russian) in Cosmic ray physics, Mathematical physics, Levy stable statistics, Monte Carlo methods with applications to anomalous processes in complex systems of various levels: from quantum dots to the Milky Way galaxy.
Fractional calculus was first developed by pure mathematicians in the middle of the 19th century. Some 100 years later, engineers and physicists have found applications for these concepts in their areas. However there has traditionally been little interaction between these two communities. In particular, typical mathematical works provide extensive findings on aspects with comparatively little significance in applications, and the engineering literature often lacks mathematical detail and precision. This book bridges the gap between the two communities. It concentrates on the class of fractional derivatives most important in applications, the Caputo operators, and provides a self-contained, thorough and mathematically rigorous study of their properties and of the corresponding differential equations. The text is a useful tool for mathematicians and researchers from the applied sciences alike. It can also be used as a basis for teaching graduate courses on fractional differential equations.
This invaluable monograph is devoted to a rapidly developing area on the research of qualitative theory of fractional ordinary and partial differential equations. It provides the readers the necessary background material required to go further into the subject and explore the rich research literature. The tools used include many classical and modern nonlinear analysis methods such as fixed point theory, measure of noncompactness method, topological degree method, the technique of Picard operators, critical point theory and semigroup theory. Based on the research work carried out by the authors and other experts during the past seven years, the contents are very recent and comprehensive. In this edition, two new topics have been added, that is, fractional impulsive differential equations, and fractional partial differential equations including fractional

Navier–Stokes equations and fractional diffusion equations.
Contents:Preliminaries:IntroductionSome Notations, Concepts and LemmasFractional CalculusSome Results from Nonlinear AnalysisSemigroupsFractional Functional Differential Equations:IntroductionNeutral Equations with Bounded Delayp-Type Neutral EquationsNeutral Equations with Infinite DelayIterative Functional Differential EquationsNotes and RemarksFractional Ordinary Differential Equations in Banach Spaces:IntroductionCauchy Problems via Measure of Noncompactness MethodCauchy Problems via Topological Degree MethodCauchy Problems via Picard Operators TechniqueNotes and RemarksFractional Abstract Evolution Equations:IntroductionEvolution Equations with Riemann–Liouville DerivativeEvolution Equations with Caputo DerivativeNonlocal Problems for Evolution EquationsAbstract Cauchy Problems with Almost Sectorial OperatorsNotes and RemarksFractional Impulsive Differential Equations:IntroductionImpulsive Initial Value ProblemsImpulsive Boundary Value ProblemsImpulsive Langevin EquationsImpulsive Evolution EquationsNotes and RemarksFractional Boundary Value Problems:IntroductionSolution for BVP with Left and Right Fractional IntegralsMultiple Solutions for BVP with ParametersInfinite Solutions for BVP with Left and Right Fractional IntegralsSolutions for BVP with Left and Right Fractional DerivativesNotes and RemarksFractional Partial Differential Equations:IntroductionFractional Navier–Stokes EquationsFractional Euler–Lagrange EquationsFractional Diffusion EquationsFractional Schrödinger EquationsNotes and Remarks
Readership: Researchers and graduate or PhD students dealing with fractional calculus and applied analysis, differential equations and related areas of research.

When a new extraordinary and outstanding theory is stated, it has to face criticism and skeptism, because it is beyond the usual concept. The fractional calculus though not new, was not discussed or developed for a long time, particularly for lack of its application to real life problems. It is extraordinary because it does not deal with ‘ordinary’ differential calculus. It is outstanding because it can now be applied to situations where existing theories fail to give satisfactory results. In this book not only mathematical abstractions are discussed in a lucid manner, with physical mathematical and geometrical explanations, but also several practical applications are given particularly for system identification, description and then efficient controls. The normal physical laws like, transport theory, electrodynamics, equation of motions, elasticity, viscosity, and several others of are based on ‘ordinary’ calculus. In this book these physical laws are generalized in fractional calculus contexts; taking, heterogeneity effect in transport background, the space having traps or islands, irregular distribution of charges, non-ideal spring with mass connected to a pointless-mass ball, material behaving with viscous as well as elastic properties, system relaxation with and without memory, physics of random delay in computer network; and several others; mapping the reality of nature closely. The concept of fractional and complex order differentiation and integration are elaborated mathematically, physically and geometrically with examples. The practical utility of local fractional differentiation for enhancing the character of singularity at phase transition or characterizing the irregularity measure of response function is deliberated. Practical results of viscoelastic experiments, fractional order controls experiments, design of fractional controller and practical circuit synthesis for fractional order elements are elaborated in this book. The book also maps theory of classical integer order differential equations to fractional calculus contexts, and deals in details with conflicting and demanding initialization issues, required in classical techniques. The book presents a modern approach to solve the 'solvable' system of fractional and other differential equations, linear, non-linear; without perturbation or transformations, but by applying physical principle of action-and-opposite-reaction, giving 'approximately exact' series solutions. Historically, Sir Isaac Newton and Gottfried Wihelm Leibniz independently discovered calculus in the middle of the 17th century. In recognition to this remarkable discovery, J.von Neumann remarked, "...the calculus was the first achievement of modern mathematics and it is difficult to overestimate its importance. I think it defines more equivocally than anything else the inception of modern mathematical analysis which is logical development, still constitute the greatest technical advance in exact thinking." This XXI century has thus started to 'think-exactly' for advancement in science & technology by growing application of fractional calculus, and this century has started speaking the language which nature understands the best.

Numerical Methods for Fractional Calculus presents numerical methods for fractional integrals and fractional derivatives, finite difference methods for fractional ordinary differential equations (FODEs) and fractional partial differential equations (FPDEs), and finite element methods for FPDEs.The book introduces the basic definitions and propertie

This book is not a text devoted to a pedagogical presentation of a specialized topic nor is it a monograph focused on the author's area of research. It accomplishes both these things while providing a rationale for why the reader ought to be interested in learning about fractional calculus. This book is for researchers who has heard about many

This is a modified version of Module 10 of the Centre for Mathematical and Statistical Sciences (CMSS). CMSS modules are notes prepared on various topics with many examples from real-life situations and exercises so that the subject matter becomes interesting to students. These modules are used for undergraduate level courses and graduate level training in various topics at CMSS. Aside from Module 8, these modules were developed by Dr A M Mathai, Director of CMSS and Emeritus Professor of Mathematics and Statistics, McGill University, Canada. Module 8 is based on the lecture notes of Professor W J Anderson of McGill University, developed for his undergraduate course

(Mathematics 447). Professor Dr Hans J Haubold has been a research collaborator of Dr A M Mathais since 1984, mainly in the areas of astrophysics, special functions and statistical distribution theory. He is also a lifetime member of CMSS and a Professor at CMSS. A large number of papers have been published jointly in these areas since 1984. The following monographs and books have been brought out in conjunction with this joint research: Modern Problems in Nuclear and Neutrino Astrophysics (A M Mathai and H J Haubold, 1988, Akademie-Verlag, Berlin); Special Functions for Applied Scientists (A MMathai and H J Haubold, 2008, Springer, New York); and The H-Function: Theory and Applications (A M Mathai, R K Saxena and H J Haubold, 2010, Springer, New York). These CMSS modules are printed at CMSS Press and published by CMSS. Copies are made available to students free of charge, and to researchers and others at production cost. For the preparation of the initial drafts of all these modules, financial assistance was made available from the Department of Science and Technology, the Government of India (DST), New Delhi under project number SR/S4/MS:287/05. Hence, the authors would like to express their thanks and gratitude to DST, the Government of India, for its financial assistance. This book is a landmark title in the continuous move from integer to non-integer in mathematics: from integer numbers to real numbers, from factorials to the gamma function, from integer-order models to models of an arbitrary order. For historical reasons, the word 'fractional' is used instead of the word 'arbitrary'. This book is written for readers who are new to the fields of fractional derivatives and fractional-order mathematical models, and feel that they need them for developing more adequate mathematical models. In this book, not only applied scientists, but also pure mathematicians will find fresh motivation for developing new methods and approaches in their fields of research. A reader will find in this book everything necessary for the initial study and immediate application of fractional derivatives fractional differential equations, including several necessary special functions, basic theory of fractional differentiation, uniqueness and existence theorems, analytical numerical methods of solution of fractional differential equations, and many inspiring examples of applications. A unique survey of many applications of fractional calculus Presents basic theory Includes a unified presentation of selected classical results, which are important for applications Provides many examples Contains a separate chapter of fractional order control systems, which opens new perspectives in control theory The first systematic consideration of Caputo's fractional derivative in comparison with other selected approaches Includes tables of fractional derivatives, which can be used for evaluation of all considered types of fractional derivatives Commences with the historical development of fractional calculus, its mathematical theory—particularly the Riemann-Liouville version. Numerous examples and theoretical applications of the theory are presented. Features topics associated with fractional differential equations. Discusses Weyl fractional calculus and some of its uses. Includes selected physical problems which lead to

fractional differential or integral equations.
Fractional calculus is a rapidly growing field of research, at the interface between probability, differential equations, and mathematical physics. It is used to model anomalous diffusion, in which a cloud of particles spreads in a different manner than traditional diffusion. This monograph develops the basic theory of fractional calculus and anomalous diffusion, from the point of view of probability. In this book, we will see how fractional calculus and anomalous diffusion can be understood at a deep and intuitive level, using ideas from probability. It covers basic limit theorems for random variables and random vectors with heavy tails. This includes regular variation, triangular arrays, infinitely divisible laws, random walks, and stochastic process convergence in the Skorokhod topology. The basic ideas of fractional calculus and anomalous diffusion are closely connected with heavy tail limit theorems. Heavy tails are applied in finance, insurance, physics, geophysics, cell biology, ecology, medicine, and computer engineering. The goal of this book is to prepare graduate students in probability for research in the area of fractional calculus, anomalous diffusion, and heavy tails. Many interesting problems in this area remain open. This book will guide the motivated reader to understand the essential background needed to read and unerstand current research papers, and to gain the insights and techniques needed to begin making their own contributions to this rapidly growing field.
Fractional equations and models play an essential part in the description of anomalous dynamics in complex systems. Recent developments in the modeling of various physical, chemical and biological systems have clearly shown that fractional calculus is not just an exotic mathematical theory, as it might have once seemed. The present book seeks to demonstrate this using various examples of equations and models with fractional and generalized operators. Intended for students and researchers in mathematics, physics, chemistry, biology and engineering, it systematically offers a wealth of useful tools for fractional calculus.
This book explains the essentials of fractional calculus and demonstrates its application in control system modeling, analysis and design. It presents original research to find high-precision solutions to fractional-order differentiations and differential equations. Numerical algorithms and their implementations are proposed to analyze multivariable fractional-order control systems. Through high-quality MATLAB programs, it provides engineers and applied mathematicians with theoretical and numerical tools to design control systems. Contents Introduction to fractional calculus and fractional-order control Mathematical prerequisites Definitions and computation algorithms of fractional-order derivatives and Integrals Solutions of linear fractional-order differential equations Approximation of fractional-order operators Modelling and analysis of multivariable fractional-order transfer function Matrices State space modelling and analysis of linear fractional-order Systems Numerical solutions of nonlinear fractional-order differential Equations Design of fractional-order PID controllers Frequency domain controller design for multivariable fractional-order Systems Inverse Laplace transforms involving fractional and irrational Operations FOTF Toolbox functions and models Benchmark problems for the assessment of fractional-order differential equation algorithms
This book presents a concise and insightful view of the knowledge on fractional-order

electrical circuits, which belongs to the subject of Electric Engineering and involves mathematics of fractional calculus. It offers an overview of fractional calculus and then describes and analyzes the basic theories and properties of fractional-order elements and fractional-order electrical circuit composed of fractional-order elements. Therein, the fundamental theorems, time-domain analysis, steady-state analysis, complex frequency domain analysis and state variable analysis of fractional-order electrical circuit are included. The fractional-order two-port networks and generalized fractional-order linear electrical circuits are also mentioned. Therefore, this book provides readers with enough background and understanding to go deeper into the topic of fractional-order electrical circuit, so that it is useful as a textbook for courses related to fractional-order elements, fractional-order electrical circuits, etc. This book is intended for students without an extensive mathematical background and is suitable for advanced undergraduate and graduate students, engineers and researchers who focus on the fractional-order elements, electrical circuits and systems.

This volume presents recent developments in the fractional calculus of functions of one and several real variables, and shows the relation of this field to a variety of areas in pure and applied mathematics. Beyond some basic properties of fractional integrals in one and many dimensions, it contains a mathematical theory of certain important weakly singular integral equations of the first kind arising in mechanics, diffraction theory and other areas of mathematical physics. The author focuses on explicit inversion formulae that can be obtained by making use of the classical Marchaudís approach and its generalization, leading to wavelet type representations.

This monograph provides a comprehensive overview of the author's work on the fields of fractional calculus and waves in linear viscoelastic media, which includes his pioneering contributions on the applications of special functions of the Mittag-Leffler and Wright types. It is intended to serve as a general introduction to the above-mentioned areas of mathematical modeling. The explanations in the book are detailed enough to capture the interest of the curious reader, and complete enough to provide the necessary background material needed to delve further into the subject and explore the research literature given in the huge general bibliography. This book is likely to be of interest to applied scientists and engineers. Contents:Essentials of Fractional CalculusEssentials of Linear ViscoelasticityFractional Viscoelastic ModelsWaves in Linear Viscoelastic Media: Dispersion and DissipationWaves in Linear Viscoelastic Media: Asymptotic RepresentationsDiffusion and Wave–Propagation via Fractional CalculusAppendices:The Eulerian FunctionsThe Bessel FunctionsThe Error FunctionsThe Exponential Integral FunctionsThe Mittag-Leffler FunctionsThe Wright Functions Readership: Graduate and PhD students in applied mathematics, classical physics, mechanical engineering and chemical physics; academic institutions; research centers. Keywords:Fractional Calculus;Fractional Derivatives;Fractional Integrals;Linear Viscoelasticity;Rheological Models;Special Functions;Mittag-Leffler Functions;Wright Functions;Integral Transforms;Laplace Transforms;Fourier Transforms;Waves;Dispersion;Dissipation;Diffusion;Anomalous DiffusionKey Features:Contains accessible mathematical language for easy understandingFeatures ample examples to reiterate concepts in the bookMakes extensive use of graphical imagesIncludes a large and informative general bibliography for further research

This work aims to present, in a systematic manner, results including the existence and uniqueness of solutions for the Cauchy Type and Cauchy problems involving nonlinear ordinary fractional differential equations.

FRACTIONAL CALCULUS: Theory and Applications deals with differentiation and integration of arbitrary order. The origin of this subject can be traced back to the end of seventeenth century, the time when Newton and Leibniz developed foundations of differential and integral calculus. Nonetheless, utility and applicability of FC to various branches of science and engineering have been realized only in last few decades. Recent years have witnessed tremendous upsurge in research activities related to the applications of FC in modeling of real-world systems. Unlike the derivatives of integral order, the non-local nature of fractional derivatives correctly models many natural phenomena containing long memory and give more accurate description than their integer counterparts.The present book comprises of contributions from academicians and leading researchers and gives a panoramic overview of various aspects of this subject: Introduction to Fractional Calculus Fractional Differential Equations Fractional Ordered Dynamical Systems Fractional Operators on Fractals Local Fractional Derivatives Fractional Control Systems Fractional Operators and Statistical Distributions Applications to Engineering

This book aims to establish a foundation for fractional derivatives and fractional differential equations. The theory of fractional derivatives enables considering any positive order of differentiation. The history of research in this field is very long, with its origins dating back to Leibniz. Since then, many great mathematicians, such as Abel, have made contributions that cover not only theoretical aspects but also physical applications of fractional calculus. The fractional partial differential equations govern phenomena depending both on spatial and time variables and require more subtle treatments. Moreover, fractional partial differential equations are highly demanded model equations for solving real-world problems such as the anomalous diffusion in heterogeneous media. The studies of fractional partial differential equations have continued to expand explosively. However we observe that available mathematical theory for fractional partial differential equations is not still complete. In particular, operator-theoretical approaches are indispensable for some generalized categories of solutions such as weak solutions, but feasible operator-theoretic foundations for wide applications are not available in monographs. To make this monograph more readable, we are restricting it to a few fundamental types of time-fractional partial differential equations, forgoing many other important and exciting topics such as stability for nonlinear problems. However, we believe that this book works well as an introduction to mathematical research in such vast fields.

This book is an unique integrated treatise, on the concepts of fractional calculus as models with applications in hydrology, soil science and geomechanics. The models are primarily fractional partial differential equations (fPDEs), and in limited cases, fractional differential equations (fDEs). It develops and applies relevant fPDEs and fDEs mainly to water flow and solute transport in porous media and overland, and in some cases, to concurrent flow and energy transfer. It is an integrated resource with theory and applications for those interested in hydrology, hydraulics and fluid mechanics. The self-contained book summaries the fundamentals for porous media and essential mathematics with extensive

references supporting the development of the model and applications.
This graduate textbook provides a self-contained introduction to modern mathematical theory on fractional differential equations. It addresses both ordinary and partial differential equations with a focus on detailed solution theory, especially regularity theory under realistic assumptions on the problem data. The text includes an extensive bibliography, application-driven modeling, extensive exercises, and graphic illustrations throughout to complement its comprehensive presentation of the field. It is recommended for graduate students and researchers in applied and computational mathematics, particularly applied analysis, numerical analysis and inverse problems.
This invaluable book provides a broad introduction to the fascinating and beautiful subject of Fractional Calculus of Variations (FCV). In 1996, FVC evolved in order to better describe non-conservative systems in mechanics. The inclusion of non-conservatism is extremely important from the point of view of applications. Forces that do not store energy are always present in real systems. They remove energy from the systems and, as a consequence, Noether's conservation laws cease to be valid. However, it is still possible to obtain the validity of Noether's principle using FCV. The new theory provides a more realistic approach to physics, allowing us to consider non-conservative systems in a natural way. The authors prove the necessary Euler–Lagrange conditions and corresponding Noether theorems for several types of fractional variational problems, with and without constraints, using Lagrangian and Hamiltonian formalisms. Sufficient optimality conditions are also obtained under convexity, and Leitmann's direct method is discussed within the framework of FCV. The book is self-contained and unified in presentation. It may be used as an advanced textbook by graduate students and ambitious undergraduates in mathematics and mechanics. It provides an opportunity for an introduction to FCV for experienced researchers. The explanations in the book are detailed, in order to capture the interest of the curious reader, and the book provides the necessary background material required to go further into the subject and explore the rich research literature.
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formalisms. Sufficient optimality conditions are also obtained under convexity, and Leitmann's direct method is discussed within the framework of FCV. The book is self-contained and unified in presentation. It may be used as an advanced textbook by graduate students and ambitious undergraduates in mathematics and mechanics. It provides an opportunity for an introduction to FCV for experienced researchers. The explanations in the book are detailed, in order to capture the interest of the curious reader, and the book provides the necessary background material required to go further into the subject and explore the rich research literature.

This book provides a broad overview of the latest developments in fractional calculus and fractional differential equations (FDEs) with an aim to motivate the readers to venture into these areas. It also presents original research describing the fractional operators of variable order, fractional-order delay differential equations, chaos and related phenomena in detail. Selected results on the stability of solutions of nonlinear dynamical systems of the non-commensurate fractional order have also been included. Furthermore, artificial neural network and fractional differential equations are elaborated on; and new transform methods (for example, Sumudu methods) and how they can be employed to solve fractional partial differential equations are discussed. The book covers the latest research on a variety of topics, including: comparison of various numerical methods for solving FDEs, the Adomian decomposition method and its applications to fractional versions of the classical Poisson processes, variable-order fractional operators, fractional variational principles, fractional delay differential equations, fractional-order dynamical systems and stability analysis, inequalities and comparison theorems in FDEs, artificial neural network approximation for fractional operators, and new transform methods for solving partial FDEs. Given its scope and level of detail, the book will be an invaluable asset for researchers working in these areas.